A Manual of Interest and Annuities

Edward Smyth

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A MANUAL

OF

INTEREST AND ANNUITIES,

COMPRISING A

POPULAR EXPLANATION

OF THE SOLUTION OF QUESTIONS OF

Compound Interest and Annuities for Years;

WITH AN AUXILIARY TABLE FOR

FIFTY-FOUR RATES OF INTEREST:

ALSO

THE VALUES OF LIFE ANNUITIES BY THE ENGLISH LIFE TABLE;

AND AN APPENDIX

CONTAINING SUGGESTIONS FOR THE MORE EQUITABLE ASSESSMENT OF

THE INCOME-TAX.

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PREFACE.

THE following pages, whilst giving facilities for calculations of compound interest, and the values of annuities for years, at rates of interest for which there are no tables published, are designed to afford information in the most simple form as to the leading principles which govern the terms for the transfer of property, especially the many varieties of it in which an interest is possessed for only a certain number of years. For want of this information, even the mechanical use of tables is very often attended with considerable embarrassment and risk.

With a view to rendering the book complete in itself, and yet restricting its size, a small table of seven-figure logarithms (with practical introduction) has been inserted; but whilst this is sufficient for a vast majority of questions upon the above subjects, it is accompanied by rules, conjectured to be new, which have been devised for its easy and accurate expansion to such extent as may be required for any purpose

in the absence of extensive, and therefore bulky tables. An Auxiliary Table has been constructed, giving numbers and logarithms necessary for the solution in a very few figures of questions at any of fifty-four rates of interest, including those which property is assumed to pay when its value in perpetuity is estimated at any integral number of years' purchase, from 20 to 33 inclusive. It is believed that the explanations which follow the examples of the several rules will be more generally serviceable than demonstrations expressed algebraically.

The values of annuities for a life, taken from the Registrar-General's Twelfth Annual Report, claim especial notice on account of the extensive range of observation upon which they are founded.

The Appendix contains suggestions for a more equitable assessment of the Income Tax, deduced from considerations arising upon the previous subjects. The question has not been approached without a consciousness of the difficulties which it involves.

E. S.

15, Harpur Street, Bloomsbury.
January, 1860.

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A MANUAL

OF

INTEREST AND ANNUITIES.

PRACTICAL INTRODUCTION TO THE LOGARITHMS.

IN the common, or Briggs's, system of logarithms the numbers in the arithmetical series—

0	1	2	3	4	5	&c.

are the respective logarithms of the numbers in the following geometrical series—

1	10	100	1000	10000	100000	&c.

Hence the logarithm of any intermediate natural number will be that of the next less number in the second series, with the addition of a certain decimal fraction, such fraction being styled the *mantissa* of the logarithm. Thus, the logarithm of 1 being 0 and that of 10 being 1. the logarithm of 6 (to no further than seven decimal places) is 0·7781513 or ·7781513.

The product of two numbers may be ascertained by adding together the logarithms of the numbers; the sum will be the logarithm of the product. Hence any power of a number may

be ascertained by multiplying the logarithm of the number by the index of the power; the product will be the logarithm of the power. Again: the quotient of two numbers may be ascertained by subtracting the logarithm of the one from the logarithm of the other; the remainder will be the logarithm of the quotient. Hence any root of a number may be ascertained by dividing the logarithm of the number by the index of the root; the quotient will be the logarithm of the root. The above series are sufficiently extensive to afford practical illustrations of these facts.

In this system of logarithms, the *mantissa* of the logarithm of any one group of figures is always the same, whether the figures denote a decimal only, or an integral or whole number only, or partly each; also the addition of ciphers to either end of the group of figures does not affect the *mantissa*, but only varies the characteristic or index of the logarithm, *i.e.*, the part to the left of the decimal point. The reason may be thus exemplified. The logarithm of 6·5 being ·8129134, that of 65, or 6·5 multiplied by 10, will be ·8129134 + 1, *i.e.*, 1·8129134; and that of 650, or 6·5 multiplied by 100, will be ·8129134 + 2, *i.e.*, 2·8129134. Again: the logarithm of

.65, or 6·5 divided by 10, will be ·8129134 − 1, which is expressed $\bar{1}$·8129134; and that of ·065, or 6·5 divided by 100, will be ·8129134 − 2, which is expressed $\bar{2}$·8129134. Instead of employing "negative characteristics" (indicated by a stroke placed above the figure, as in the last two instances,) it somewhat simplifies a calculation to augment such characteristics by 10 in order to render them positive, every 10 so imported into the calculation being rejected at some subsequent stage. Thus, the logarithms 9·8129134 and 8·8129134 might have been respectively substituted for $\bar{1}$·8129134 and $\bar{2}$·8129134, the characteristics in the former cases being in fact numbers which are the complements to 10 of those in the latter. The following graduated examples will serve as a recapitulation :—

Log. of	8010	is 3·9036325	
„	801	is 2·9036325	
„	80·1	is 1·9036325	
„	8·01	is ·9036325	
„	·801	is $\bar{1}$·9036325	or 9·9036325
„	·0801	is $\bar{2}$·9036325	or 8·9036325

Hence if the given number be either integral or mixed, the characteristic of its logarithm will always be a number less by unity than the

number of figures comprised in the integral portion of the given number. If the given number be entirely decimal, the characteristic will be negative, and (when expressed accordingly, *i.e.*, without augmentation) will always be a number greater by unity than the number of ciphers commencing the decimal.

The following table includes the *mantissa* of each triplet of figures; and with a view to rendering it serviceable for numbers comprising more than three figures, the methods about to be described are now proposed. One circumstance must, however, be first mentioned. The table gives the logarithms to the extent of seven decimal places. Therefore whenever the eighth figure would be 5 or more, the actual seventh figure is increased by 1; in other cases the eighth and following figures are simply rejected. This obvious plan of modifying a figure, by way of compromise for the rejection of any figures that follow it, must be pursued (whether mentioned or not) throughout the calculations by all the rules in the following pages, and it is observable in the examples. Wherever it has been deemed advisable to allude to the circumstance, the word "modified" or "compromised" has been employed for the sake of brevity.

Methods of finding from the General Table under 1000 *the logarithm of a given number to* 5, 6, *or* 7 *decimal figures respectively.*

NOTE. For the reason explained in the Introduction, ciphers at either end of a given number are to be disregarded in the following rules except in fixing the characteristic.

Case I. If the first figure of the given number

exceed 1 when 5 decimal figures are required ;
or exceed 5 when 6 decimal figures are required ;
or be 9 when 7 decimal figures are required ;

Look in the general table for the "mantissa" corresponding to the first three figures of the given number, and add to it in respect of the remaining figures a proportionate part of the annexed "difference." To the total prefix the requisite characteristic upon the principle explained in the Introduction.

The above-mentioned proportionate part of the difference must be estimated by multiplying the difference by the said remaining figures of the given number, and rejecting from the right hand of the product as many figures as remained in the given number. The detail of this operation is shortened in the examples by a course which is sufficiently obvious.

Case II.—With respect to all numbers not embraced by Case I. :—Turn to the

First Part of the Preliminary Table, when 5 decimals are required ;

or to the Second Part, when 6 decimals are required ;

or to the Third Part, when 7 decimals are required ;

and look for such "Number" as is next higher than the commencing figure or figures of the given number, and for the corresponding "mantissa." Divide the given number by the number so selected, and find, as in Case I., the mantissa corresponding to the quotient ; add to this the mantissa selected from the Preliminary Table. Reject from the total the unit appearing as a characteristic, and substitute the requisite figure upon the principle explained in the Introduction.

When the number in the Third Part of the Preliminary Table happens to be more than 12, its two factors should be employed in the order of their appearance in such Table, and ciphers may be assumed to be appended to the given number. The quotient produced upon the division by the second factor need not be carried beyond a seventh figure (modified), if it would extend so far; but it can in general be advantageously terminated within the seventh figure by the use of a vulgar fraction, the factors firstly named in the Preliminary Table having been selected with this view.

Ex.—Required the log. to 6 decimals of 7153.

715	8543060
6070 × 3 =	1821,0
	8544881

3·854488 is the log. required.

Ex.—Required the log. to 6 decimals of 281·484.

3) 281484	3	4771213
93828	938	9722028
	4628 × 2 =	925,6
	" × 8 =	370,24
		4494537

2·449454 is the log. required; but 2·4494537 is correct to the seventh place, and it may be seen that the calculation would have been the same if the requirement of the question had extended to 7 decimals.

Ex.—Required the log. to 7 decimals of ·016048.

2) 16048	18	2552725
9) 8024	891	9498777
$891\frac{5}{9}$	$4872 \times \frac{5}{9} =$	2707
		2054209

2·2054209 (or 8·2054209) is the log. required.

It can happen very rarely that the foregoing methods will fail to produce a correct modification of the final figure of their respective results, excepting in the case of the seventh-figure method, which will occasionally err by a unit in that figure.

N.B.—The utmost possible amount of error that can exist in the *seven* figures of a mantissa resulting from the five or the six-figure methods, is less than the equivalent of a variation in the given number to the extent of, respectively, only its quarter-millionth or its millionth part; the error will generally be very much less.

Methods of finding from the General Table under 1000 *the number corresponding to a given logarithm to either* 4, 5, 6, *or* 7 *figures respectively.*

NOTE. The numbers obtained by the following methods will be either integral, decimal, or mixed, according to the characteristics of the given logarithms, as explained in the Introduction.

Case I. If the number be only required to

4 figures
or to 5 figures, if the mantissa exceed 3010300;
or to 6 ditto 7781513;
or to 7 ditto 9542425;

From the given mantissa subtract the next lower in the General Table, and observe its "Number" and "Difference." This number will constitute the first three figures of the required number. To the right of the remainder left upon the above subtraction annex as many ciphers as there are figures yet to be ascertained in the required number. Divide this augmented remainder by the difference found in the General Table, and annex the quotient to the right of the three figures above named.

Case II. With respect to all logarithms not embraced by Case I:—Refer to the First Part of the Preliminary Table, when the number is required to 5 figures;
or to the Second Part when required to 6 figures,
or to the Third Part ditto 7 figures,
and select from it the "Mantissa" next higher than the mantissa of the given logarithm, and observe the corresponding "Number." Prefix a unit to the latter mantissa, and then deduct from it the former. Find as in Case I. the number cor-

responding to the remainder, and multiply it by the number selected from the Preliminary Table.

Ex.—Of what number to 5 figures is 4·0104865 the log.?

	1·0104865	
	3010300 *i.e.* 2	
	7094565	51222
	7092700 *i.e.* 512	2
8474)	186500(22	102444

10244 is the number required.

Ex.—Of what number to 6 figures is 3·7788998 the log.?

	7788998
	7788745 *i.e.* 601
722,0)	25300,0(035

6010·35 is the number required.

Ex.—Of what number to 7 figures is $\bar{1}$·6360819 the log.?

	1·6360819
	6532125 *i.e.* 45
	9828694
	9827234 *i.e.* 961
4517)	14600000(3232

9613232 × 45 = 432595440

·4325954 is the number required.

It can happen very rarely that these methods of finding a number will fail to produce a correct modification of the final figure of their respective results.

First Part.		Second Part.	
No.	Mantissa.	No.	Mantissa.
2	3010300	12	0791812
		2	3010300
		3	4771213
		4	6020600
		5	6989700
		6	7781513

Third Part.					
No.	Factors.	Mantissa.	No.	Factors.	Mantissa.
11		0413927	35	5 7	5440680
12		0791812	36	4 9	5563025
132	12 11	1205739	4		6020600
14	2 7	1461280	42	6 7	6232493
15	5 3	1760913	45	5 9	6532125
16	2 8	2041200	5		6989700
18	2 9	2552725	55	5 11	7403627
2		3010300	6		7781513
22	2 11	3424227	64	8 8	8061800
24	2 12	3802112	7		8450980
25	5 5	3979400	72	8 9	8573325
27	3 9	4313638	8		9030900
3		4771213	81	9 9	9084850
32	4 8	5051500	9		9542425

END OF PRELIMINARY TABLE.

GENERAL TABLE

[illegible]

LOGARITHMS.

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
100	0000000	43214	125	0969100	34605
101	0043214	42788	126	1003705	34332
102	0086002	42370	127	1038037	34063
103	0128372	41961	128	1072100	33797
104	0170333	41560	129	1105897	33537
105	0211893	41166	130	1139434	33279
106	0253059	40779	131	1172713	33026
107	0293838	40400	132	1205739	32777
108	0334238	40027	133	1238516	32532
109	0374265	39662	134	1271048	32290
110	0413927	39303	135	1303338	32051
111	0453230	38950	136	1335389	31817
112	0492180	38604	137	1367206	31585
113	0530784	38265	138	1398791	31357
114	0569049	37929	139	1430148	31132
115	0606978	37602	140	1461280	30911
116	0644580	37279	141	1492191	30692
117	0681859	36961	142	1522883	30477
118	0718820	36650	143	1553360	30265
119	0755470	36342	144	1583625	30055
120	0791812	36042	145	1613680	29849
121	0827854	35744	146	1643529	29644
122	0863598	35453	147	1673173	29444
123	0899051	35166	148	1702617	29246
124	0934217	34883	149	1731863	29050

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
150	1760913	28856	175	2430380	24747
151	1789769	28667	176	2455127	24606
152	1818436	28478	177	2479733	24467
153	1846914	28293	178	2504200	24330
154	1875207	28110	179	2528530	24195
155	1903317	27929	**180**	2552725	24061
156	1931246	27751	181	2576786	23928
157	1958997	27574	182	2600714	23797
158	1986571	27400	183	2624511	23667
159	2013971	27229	184	2648178	23539
160	2041200	27059	185	2671717	23412
161	2068259	26891	186	2695129	23287
162	2095150	26726	187	2718416	23162
163	2121876	26562	188	2741578	23040
164	2148438	26401	189	2764618	22918
165	2174839	26242	**190**	2787536	22798
166	2201081	26084	191	2810334	22678
167	2227165	25928	192	2833012	22561
168	2253093	25774	193	2855573	22444
169	2278867	25622	194	2878017	22329
170	2304489	25472	195	2900346	22215
171	2329961	25323	196	2922561	22101
172	2355284	25177	197	2944662	21990
173	2380461	25031	198	2966652	21879
174	2405492	24888	199	2988531	21769

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
200	3010300	21661	225	3521825	19259
201	3031961	21553	226	3541084	19175
202	3053514	21446	227	3560259	19089
203	3074960	21342	228	3579348	19007
204	3096302	21237	229	3598355	18923
205	3117539	21133	**230**	3617278	18842
206	3138672	21031	231	3636120	18760
207	3159703	20930	232	3654880	18679
208	3180633	20830	233	3673559	18600
209	3201463	20730	234	3692159	18520
210	3222193	20632	235	3710679	18441
211	3242825	20534	236	3729120	18363
212	3263359	20437	237	3747483	18287
213	3283796	20342	238	3765770	18209
214	3304138	20247	239	3783979	18133
215	3324385	20153	**240**	3802112	18058
216	3344538	20059	241	3820170	17984
217	3364597	19968	242	3838154	17909
218	3384565	19876	243	3856063	17835
219	3404441	19786	244	3873898	17763
220	3424227	19696	245	3891661	17690
221	3443923	19607	246	3909351	17619
222	3463530	19519	247	3926970	17547
223	3483049	19431	248	3944517	17476
224	3502480	19345	249	3961993	17407

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
250	3979400	17337	275	4393327	15764
251	3996737	17268	276	4409091	15707
252	4014005	17200	277	4424798	15650
253	4031205	17132	278	4440448	15594
254	4048337	17065	279	4456042	15538
255	4065402	16998	280	4471580	15483
256	4082400	16931	281	4487063	15428
257	4099331	16866	282	4502491	15373
258	4116197	16801	283	4517864	15319
259	4132998	16735	284	4533183	15266
260	4149733	16672	285	4548449	15211
261	4166405	16608	286	4563660	15159
262	4183013	16544	287	4578819	15106
263	4199557	16482	288	4593925	15053
264	4216039	16420	289	4608978	15002
265	4232459	16357	290	4623980	14950
266	4248816	16297	291	4638930	14899
267	4265113	16235	292	4653829	14847
268	4281348	16175	293	4668676	14797
269	4297523	16115	294	4683473	14747
270	4313638	16055	295	4698220	14697
271	4329693	15996	296	4712917	14647
272	4345689	15937	297	4727564	14599
273	4361626	15880	298	4742163	14549
274	4377506	15821	299	4756712	14501

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
300	4771213	14452	325	5118834	13342
301	4785665	14404	326	5132176	13302
302	4800069	14357	327	5145478	13260
303	4814426	14310	328	5158738	13221
304	4828736	14262	329	5171959	13180
305	4842998	14216	330	5185139	13141
306	4857214	14170	331	5198280	13101
307	4871384	14123	332	5211381	13061
308	4885507	14078	333	5224442	13023
309	4899585	14032	334	5237465	12983
310	4913617	13987	335	5250448	12945
311	4927604	13942	336	5263393	12906
312	4941546	13897	337	5276299	12868
313	4955443	13853	338	5289167	12830
314	4969296	13810	339	5301997	12792
315	4983106	13765	340	5314789	12755
316	4996871	13722	341	5327544	12717
317	5010593	13678	342	5340261	12680
318	5024271	13636	343	5352941	12643
319	5037907	13593	344	5365584	12607
320	5051500	13550	345	5378191	12570
321	5065050	13509	346	5390761	12534
322	5078559	13466	347	5403295	12497
323	5092025	13425	348	5415792	12462
324	5105450	13384	349	5428254	12426

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
350	5440680	12391	375	5740313	11565
351	5453071	12356	376	5751878	11536
352	5465427	12320	377	5763414	11504
353	5477747	12286	378	5774918	11474
354	5490033	12251	379	5786392	11444
355	5502284	12216	**380**	5797836	11414
356	5514500	12182	381	5809250	11384
357	5526682	12148	382	5820634	11354
358	5538830	12114	383	5831988	11324
359	5550944	12081	384	5843312	11295
360	5563025	12047	385	5854607	11266
361	5575072	12014	386	5865873	11237
362	5587086	11980	387	5877110	11207
363	5599066	11948	388	5888317	11179
364	5611014	11915	389	5899496	11150
365	5622929	11882	**390**	5910646	11122
366	5634811	11850	391	5921768	11093
367	5646661	11817	392	5932861	11065
368	5658478	11786	393	5943926	11036
369	5670264	11753	394	5954962	11009
370	5682017	11722	395	5965971	10981
371	5693739	11690	396	5976952	10953
372	5705429	11659	397	5987905	10926
373	5717088	11628	398	5998831	10898
374	5728716	11597	399	6009729	10871

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
400	6020600	10844	425	6283889	10207
401	6031444	10817	426	6294096	10183
402	6042261	10789	427	6304279	10159
403	6053050	10764	428	6314438	10135
404	6063814	10736	429	6324573	10112
405	6074550	10710	430	6334685	10088
406	6085260	10684	431	6344773	10064
407	6095944	10658	432	6354837	10042
408	6106602	10631	433	6364879	10018
409	6117233	10606	434	6374897	9996
410	6127839	10579	435	6384893	9972
411	6138418	10554	436	6394865	9949
412	6148972	10529	437	6404814	9927
413	6159501	10502	438	6414741	9904
414	6170003	10478	439	6424645	9882
415	6180481	10452	440	6434527	9859
416	6190933	10428	441	6444386	9837
417	6201361	10402	442	6454223	9814
418	6211763	10377	443	6464037	9793
419	6222140	10353	444	6473830	9770
420	6232493	10328	445	6483600	9749
421	6242821	10304	446	6493349	9726
422	6253125	10279	447	6503075	9705
423	6263404	10255	448	6512780	9683
424	6273659	10230	449	6522463	9662

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
450	6532125	9640	475	6766936	9134
451	6541765	9619	476	6776070	9114
452	6551384	9598	477	6785184	9095
453	6560982	9577	478	6794279	9076
454	6570559	9555	479	6803355	9057
455	6580114	9534	480	6812412	9039
456	6589648	9514	481	6821451	9019
457	6599162	9493	482	6830470	9001
458	6608655	9472	483	6839471	8983
459	6618127	9451	484	6848454	8963
460	6627578	9431	485	6857417	8946
461	6637009	9411	486	6866363	8927
462	6646420	9390	487	6875290	8908
463	6655810	9370	488	6884198	8891
464	6665180	9350	489	6893089	8872
465	6674530	9329	490	6901961	8854
466	6683859	9310	491	6910815	8836
467	6693169	9290	492	6919651	8818
468	6702459	9269	493	6928469	8800
469	6711728	9251	494	6937269	8783
470	6720979	9230	495	6946052	8765
471	6730209	9211	496	6954817	8747
472	6739420	9191	497	6963564	8729
473	6748611	9172	498	6972293	8712
474	6757783	9153	499	6981005	8695

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
500	6989700	8677	525	7201593	8264
501	6998377	8660	526	7209857	8249
502	7007037	8643	527	7218106	8233
503	7015680	8625	528	7226339	8218
504	7024305	8609	529	7234557	8202
505	7032914	8591	530	7242759	8186
506	7041505	8575	531	7250945	8171
507	7050080	8557	532	7259116	8156
508	7058637	8541	533	7267272	8141
509	7067178	8524	534	7275413	8125
510	7075702	8507	535	7283538	8110
511	7084209	8491	536	7291648	8095
512	7092700	8474	537	7299743	8080
513	7101174	8457	538	7307823	8065
514	7109631	8441	539	7315888	8050
515	7118072	8425	540	7323938	8035
516	7126497	8408	541	7331973	8020
517	7134905	8393	542	7339993	8005
518	7143298	8376	543	7347998	7991
519	7151674	8359	544	7355989	7976
520	7160033	8344	545	7363965	7961
521	7168377	8328	546	7371926	7947
522	7176705	8312	547	7379873	7933
523	7185017	8296	548	7387806	7917
524	7193313	8280	549	7395723	7904

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
550	7403627	7889	575	7596678	7547
551	7411516	7875	576	7604225	7533
552	7419391	7860	577	7611758	7520
553	7427251	7847	578	7619278	7508
554	7435098	7832	579	7626786	7494
555	7442930	7818	580	7634280	7481
556	7450748	7804	581	7641761	7469
557	7458552	7790	582	7649230	7456
558	7466342	7776	583	7656686	7442
559	7474118	7762	584	7664128	7431
560	7481880	7749	585	7671559	7417
561	7489629	7734	586	7678976	7405
562	7497363	7721	587	7686381	7392
563	7505084	7707	588	7693773	7380
564	7512791	7693	589	7701153	7367
565	7520484	7680	590	7708520	7355
566	7528164	7667	591	7715875	7342
567	7535831	7652	592	7723217	7330
568	7543483	7640	593	7730547	7317
569	7551123	7626	594	7737864	7306
570	7558749	7612	595	7745170	7293
571	7566361	7599	596	7752463	7280
572	7573960	7586	597	7759743	7269
573	7581546	7573	598	7767012	7256
574	7589119	7559	599	7774268	7245

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
600	7781513	7232	625	7958800	6943
601	7788745	7220	626	7965743	6932
602	7795965	7208	627	7972675	6921
603	7803173	7196	628	7979596	6910
604	7810369	7185	629	7986506	6899
605	7817554	7172	630	7993405	6889
606	7824726	7161	631	8000294	6877
607	7831887	7149	632	8007171	6866
608	7839036	7137	633	8014037	6856
609	7846173	7125	634	8020893	6844
610	7853298	7114	635	8027737	6834
611	7860412	7102	636	8034571	6823
612	7867514	7091	637	8041394	6813
613	7874605	7079	638	8048207	6802
614	7881684	7067	639	8055009	6791
615	7888751	7056	640	8061800	6780
616	7895807	7045	641	8068580	6770
617	7902852	7033	642	8075350	6760
618	7909885	7021	643	8082110	6749
619	7916906	7011	644	8088859	6738
620	7923917	6999	645	8095597	6728
621	7930916	6988	646	8102325	6718
622	7937904	6976	647	8109043	6707
623	7944880	6966	648	8115750	6697
624	7951846	6954	649	8122447	6687

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
650	8129134	6676	675	8293038	6429
651	8135810	6666	676	8299467	6420
652	8142476	6656	677	8305887	6410
653	8149132	6645	678	8312297	6401
654	8155777	6636	679	8318698	6391
655	8162413	6625	**680**	8325089	6382
656	8169038	6616	681	8331471	6373
657	8175654	6605	682	8337844	6363
658	8182259	6595	683	8344207	6354
659	8188854	6585	684	8350561	6345
660	8195439	6576	685	8356906	6335
661	8202015	6565	686	8363241	6326
662	8208580	6555	687	8369567	6317
663	8215135	6546	688	8375884	6308
664	8221681	6535	689	8382192	6299
665	8228216	6526	**690**	8388491	6289
666	8234742	6516	691	8394780	6281
667	8241258	6507	692	8401061	6271
668	8247765	6496	693	8407332	6263
669	8254261	6487	694	8413595	6253
670	8260748	6477	695	8419848	6244
671	8267225	6468	696	8426092	6236
672	8273693	6458	697	8432328	6226
673	8280151	6448	698	8438554	6218
674	8286599	6439	699	8444772	6208

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
700	8450980	6200	725	8603380	5986
701	8457180	6191	726	8609366	5978
702	8463371	6182	727	8615344	5970
703	8469553	6174	728	8621314	5961
704	8475727	6164	729	8627275	5954
705	8481891	6156	730	8633229	5945
706	8488047	6147	731	8639174	5937
707	8494194	6139	732	8645111	5929
708	8500333	6129	733	8651040	5921
709	8506462	6121	734	8656961	5912
710	8512583	6113	735	8662873	5905
711	8518696	6104	736	8668778	5897
712	8524800	6095	737	8674675	5889
713	8530895	6087	738	8680564	5880
714	8536982	6078	739	8686444	5873
715	8543060	6070	740	8692317	5865
716	8549130	6062	74	8698182	5857
717	8555192	6052	742	8704039	5849
718	8561244	6045	743	8709888	5841
719	8567289	6036	744	8715729	5834
720	8573325	6028	745	8721563	5825
721	8579353	6019	746	8727388	5818
722	8585372	6011	747	8733206	5810
723	8591383	6003	748	8739016	5802
724	8597386	5994	749	8744818	5795

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
750	8750613	5786	775	8893017	5600
751	8756399	5779	776	8898617	5593
752	8762178	5772	777	8904210	5586
753	8767950	5763	778	8909796	5579
754	8773713	5757	779	8915375	5571
755	8779470	5748	780	8920946	5564
756	8785218	5741	781	8926510	5558
757	8790959	5733	782	8932068	5550
758	8796692	5726	783	8937618	5543
759	8802418	5718	784	8943161	5536
760	8808136	5711	785	8948697	5528
761	8813847	5703	786	8954225	5522
762	8819550	5695	787	8959747	5515
763	8825245	5689	788	8965262	5508
764	8830934	5680	789	8970770	5501
765	8836614	5674	790	8976271	5494
766	8842288	5666	791	8981765	5487
767	8847954	5658	792	8987252	5480
768	8853612	5651	793	8992732	5473
769	8859263	5644	794	8998205	5466
770	8864907	5637	795	9003671	5460
771	8870544	5629	796	9009131	5452
772	8876173	5622	797	9014583	5446
773	8881795	5615	798	9020029	5439
774	8887410	5607	799	9025468	5432

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
800	9030900	5425	825	9164539	5261
801	9036325	5419	826	9169800	5255
802	9041744	5411	827	9175055	5248
803	9047155	5405	828	9180303	5242
804	9052560	5399	829	9185545	5236
805	9057959	5391	830	9190781	5229
806	9063350	5385	831	9196010	5223
807	9068735	5379	832	9201233	5217
808	9074114	5371	833	9206450	5211
809	9079485	5365	834	9211661	5204
810	9084850	5359	835	9216865	5198
811	9090209	5351	836	9222063	5192
812	9095560	5345	837	9227255	5185
813	9100905	5339	838	9232440	5180
814	9106244	5332	839	9237620	5173
815	9111576	5326	840	9242793	5167
816	9116902	5319	841	9247960	5161
817	9122221	5312	842	9253121	5155
818	9127533	5306	843	9258276	5148
819	9132839	5300	844	9263424	5143
820	9138139	5293	845	9268567	5137
821	9143432	5286	846	9273704	5130
822	9148718	5280	847	9278834	5125
823	9153998	5274	848	9283959	5118
824	9159272	5267	849	9289077	5112

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
850	9294189	5107	875	9420081	4960
851	9299296	5100	876	9425041	4955
852	9304396	5094	877	9429996	4949
853	9309490	5089	878	9434945	4944
854	9314579	5082	879	9439889	4938
855	9319661	5077	880	9444827	4932
856	9324738	5070	881	9449759	4927
857	9329808	5065	882	9454686	4921
858	9334873	5059	883	9459607	4916
859	9339932	5053	884	9464523	4910
860	9344985	5047	885	9469433	4904
861	9350032	5041	886	9474337	4899
862	9355073	5035	887	9479236	4894
863	9360108	5029	888	9484130	4888
864	9365137	5024	889	9489018	4882
865	9370161	5018	890	9493900	4877
866	9375179	5012	891	9498777	4872
867	9380191	5006	892	9503649	4866
868	9385197	5001	893	9508515	4860
869	9390198	4995	894	9513375	4855
870	9395193	4989	895	9518230	4850
871	9400182	4983	896	9523080	4844
872	9405165	4977	897	9527924	4839
873	9410142	4972	898	9532763	4834
874	9415114	4967	899	9537597	4828

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
900	9542425	4823	925	9661417	4693
901	9547248	4817	926	9666110	4687
902	9552065	4813	927	9670797	4683
903	9556878	4806	928	9675480	4677
904	9561684	4802	929	9680157	4672
905	9566486	4796	930	9684829	4668
906	9571282	4791	931	9689497	4662
907	9576073	4785	932	9694159	4657
908	9580858	4781	933	9698816	4653
909	9585639	4775	934	9703469	4647
910	9590414	4770	935	9708116	4642
911	9595184	4764	936	9712758	4638
912	9599948	4760	937	9717396	4632
913	9604708	4754	938	9722028	4628
914	9609462	4749	939	9726656	4623
915	9614211	4744	940	9731279	4617
916	9618955	4738	941	9735896	4613
917	9623693	4734	942	9740509	4608
918	9628427	4728	943	9745117	4603
919	9633155	4723	944	9749720	4598
920	9637878	4718	945	9754318	4593
921	9642596	4713	946	9758911	4589
922	9647309	4708	947	9763500	4583
923	9652017	4703	948	9768083	4579
924	9656720	4697	949	9772662	4574

No.	Mantissa.	Diff.	No.	Mantissa.	Diff.
950	9777236	4569	975	9890046	4452
951	9781805	4564	976	9894498	4448
952	9786369	4560	977	9898946	4443
953	9790929	4555	978	9903389	4438
954	9795484	4550	979	9907827	4434
955	9800034	4545	980	9912261	4429
956	9804579	4540	981	9916690	4425
957	9809119	4536	982	9921115	4420
958	9813655	4531	983	9925535	4416
959	9818186	4526	984	9929951	4411
960	9822712	4522	985	9934362	4407
961	9827234	4517	986	9938769	4403
962	9831751	4512	987	9943172	4397
963	9836263	4507	988	9947569	4394
964	9840770	4503	989	9951963	4389
965	9845273	4498	990	9956352	4385
966	9849771	4494	991	9960737	4380
967	9854265	4489	992	9965117	4375
968	9858754	4484	993	9969492	4372
969	9863238	4479	994	9973864	4367
970	9867717	4475	995	9978231	4362
971	9872192	4471	996	9982593	4359
972	9876663	4465	997	9986952	4353
973	9881128	4462	998	9991305	4350
974	9885590	4456	999	9995655	4345

COMPOUND INTEREST

AND

ANNUITIES FOR TERMS OF YEARS.

COMPOUND Interest is to be understood throughout the following Problems.

It is also assumed that interest is receivable punctually at the end of each year, and is itself instantly reinvested in the cases in which a re-investment is spoken of. Although the income derived from most properties is receivable oftener than once a-year, this advantage is counter-balanced by the ordinary delays in the receipts and reinvestments.

It may be here observed, that the rate of interest at which any particular property is valued, is the rate which the valuer considers it should, as capital, yield to the owner, having regard to the risks and the contingencies to which the income is liable, and also, on the other hand, to any incidental advantages which the possession of the property may afford.

It has been thought desirable in the following

rules and explanations to indicate the values of £1 under the particular circumstances, such values being those shown in published tables. The subsequent multiplication of these values, by the sums which are the subjects of the examples, might sometimes have been more rapidly performed by logarithms.

Before considering the following problems, it is necessary to understand the signification of the characteristic of a logarithm, explained in the "Practical Introduction" at the commencement of the book.

In the case of most properties, uncertainty to a greater or less extent exists as to the precise net amounts derived, or estimated, as in the case of purchases, to be derivable from them; and the proper rate of interest is also generally a matter of opinion. Therefore in finding the number corresponding to a logarithm, produced by the following rules (such number being often that which in the case of annuities is commonly called the "years' purchase" of the annual income,) it is unnecessary in a large majority of practical questions to seek the number to a greater extent than three figures, as these will include one and sometimes two places of decimals. In the examples of the following rules,

with the exception of those relating to the amounts of annuities, such a logarithm is therefore resolved into a number of only three figures, a course which simply requires an inspection of the foregoing table of the logarithms of numbers. If it were desired to ascertain the number with further accuracy, it might be obtained to four or five figures by the methods hereinbefore set out, or by the inspection of an extensive table of logarithms, should there be one at hand. If more than three figures had been employed, the results of some of the examples would have been more exact, and the illustrations appended to the explanations of some of the rules would have shown their accuracy more precisely; it will be readily understood that in order to secure final results, correct to a unit, when large sums are involved, it would be necessary in some of the problems to ascertain such numbers to four or five figures. The ultimate error, however, arising from the employment of only three figures in any case, bears a very insignificant proportion to the magnitude of the final result; and this result, so obtained, is, for the reasons before stated, sufficiently accurate for almost every transaction, it being very much nearer to the strict mathematical truth than is the result

which is likely to be realized in an actual dealing with the property concerned.

It may be mentioned that the legal and other costs, incidental to the several transactions instanced in the examples, are matters for additional consideration.

Rate per cent. interest per annum.	Logarithm of the amount of £1 in one year.	Present value of £1 per annum in perpetuity; or freehold.	Logarithm of said perpetuity.
£		£	
¼	·0010844	400·	2·6020600
½	·0021661	200·	2·3010300
¾	·0032451	133·333 +	2·1249387
1	·0043214	100·	2·0000000
1¼	·0053950	80·	1·9030900
1½	·0064660	66·667 –	1·8239087
1¾	·0075344	57·143 –	1·7569620
2	·0086002	50·	1·6989700
2¼	·0096633	44·444 +	1·6478175
2½	·0107239	40·	1·6020600
2¾	·0117818	36·364 –	1·5606673
3	·0128372	33·333 +	1·5228787
3·03030 +	·0129650	33·	1·5185139
3·125	·0133640	32·	1·5051500
3·22581 –	·0137883	31·	1·4913617
3¼	·0138901	30·769 +	1·4881166
3·33333 +	·0142404	30·	1·4771213
3·44828 –	·0147233	29·	1·4623980
3½	·0149403	28·571 +	1·4559320
3·57143 –	·0152400	28·	1·4471580
3·70370 +	·0157943	27·	1·4313638
3¾	·0159881	26·667 –	1·4259687
3·84615 +	·0163904	26·	1·4149733
4	·0170333	25·	1·3979400
4·16667 –	·0177288	24·	1·3802112
4¼	·0180761	23·529 +	1·3716111
4.34783 –	·0184834	23·	1·3617278

Rate per cent. interest per annum.	Logarithm of the amount of £1 in one year.	Present value of £1 per annum in perpetuity; or freehold.	Logarithm of said perpetuity.
£		£	
4½	·0191163	22·222 +	1·3467875
4·54545 +	·0193052	22·	1·3424227
4¾	·0201540	21·053 –	1·3233064
4·76190 +	·0202034	21·	1·3222193
5	·0211893	20·	1·3010300
5¼	·0222221	19·048 –	1·2798407
5½	·0232525	18·182 –	1·2596373
5¾	·0242804	17·391 +	1·2403322
6	·0253059	16·667 –	1·2218487
6½	·0273496	15·385 –	1·1870866
7	·0293838	14·286 –	1·1549020
7½	·0314085	13·333 +	1·1249387
8	·0334238	12·5	1·0969100
8½	·0354297	11·765 –	1·0705811
9	·0374265	11·111 +	1·0457575
9½	·0394141	10·526 +	1·0222764
10	·0413927	10·	1·0000000
11	·0453230	9·091 –	·9586073
12	·0492180	8·333 +	·9208188
13	·0530784	7·692 +	·8860566
14	·0569049	7·143 –	·8538720
15	·0606978	6·667 –	·8239087
16	·0644580	6·25	·7958800
17	·0681859	5·882 +	·7695511
18	·0718820	5·556 –	·7447275
19	·0755470	5·263 +	·7212464
20	·0791812	5·	·6989700

COMPOUND INTEREST.

Problem 1.—To find the amount of a given sum in any number of years.

Multiply the logarithm of the amount of £1 in one year at the stated rate of interest (Auxiliary Table) by the number of years; the natural number corresponding to the product will be the amount of £1 in the time. Multiply the given sum by this amount; the product will be the amount required.

Example 1.—To what sum would £140 amount in 9 years at 4 per cent. interest?

$$\begin{array}{r} .0170333 \\ 9 \\ \hline .1532997 - 1.42 \\ \hline \end{array}$$

£140 × 1.42 = £199. *Answer.*

Ex. 2.—The amount of £500 in 9 years at 8 per cent. = £500 × 2 = £1000.

Ex. 3.—The amount of £940 in 30 years at 4½ per cent. = £940 × 3.49 = £3280.

Let the rate of interest in Ex. 1, viz., 4 per cent., be taken for the sake of explanation. The amount of £1 in one year is £1, with the addition of one year's interest upon it; together £1.04. The amount of £1 in two years is therefore £1.04 with the addition of the interest upon this during the second year. This amount will therefore be ascertained by multiplying £1.04 by 1.04;* consequently the amount of £1 in two years is the square or second power of its amount in one year, viz., £1.0816. The amount of £1 in three years is therefore £1.0816, with the addition of the interest upon this during the third year. This amount will therefore be ascertained by multiplying £1.0816 by 1.04; consequently the amount of £1 in three years is the cube or third power of its amount in one year, viz., £1.124864. And so on for every succeeding year. Therefore the amount of £1 in any given number of years is the amount of £1 in one year raised to a power of which the index is the same as the number of years; or if

* It is evident that the multiplying by 1·04 is an operation which embraces the calculation of one year's interest and the addition of the principal.

the operation is to be performed by logarithms (and otherwise it would be most laborious) the method will, as may be gathered from the "Practical Introduction," be to multiply the logarithm of the amount of £1 in one year by the number of years, the product being the logarithm of the power, i.e., the amount required.

PROBLEM 2.—To find the present value of a sum due at the end of any number of years,

Find (as in Prob. 1) the amount of £1 in the time, thus:—Multiply the log. of the amount of £1 in one year at the stated rate of interest (Auxiliary Table) by the number of years; the natural number corresponding to the product will be the amount of £1 in the time. Divide the given sum by this amount; the quotient will be the present value required (A). This division might be performed by subtracting the respective logarithms (B).

The present value of £1 due at the end of any number of years might be similarly ascertained. Then by multiplying the given sum by such present value, the result obtained will be the same as before (C).

Ex. 4.—What is the present value at $3\frac{1}{2}$ per cent.

of £1200 receivable at the end of 30 years?

Or:—A property is valued as freehold at £1200. It is outstanding upon a beneficial* lease for 30 years. Upon a sale of the interest of the reversioner, what sum should he obtain in order that he may, if he invest it at 3½ per cent., provide himself at the end of the 30 years with the fee simple value of the property, viz., £1200? (In this case an additional sum should be obtained in respect of the present value of any rent receivable by the reversioner during the continuance of the lease; it may be ascertained by the rule for Prob. 12.)

(A)

$$
\begin{array}{r}
\cdot 0138901 \\
30 \\
\hline
\cdot 4167030 = 2\cdot 61 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
2\cdot 61)\ 1200 \quad (£460 \quad \textit{Ans.} \\
\qquad\ \ 1044 \\
\qquad\ \ \overline{\quad\ 15600}
\end{array}
$$

* A lease of property is said to be beneficial when it is granted at a rent less than the actual annual value, compensation being made by the payment by the lessee (at the time of taking the lease) of a sum of money called a premium or fine calculated as in Problems 12, 16, &c.

(B)

$$
\begin{array}{r}
\cdot0138901 \\
30 \\
\hline
\cdot4167030 \\
\text{Log. } 1200 = 3\cdot0791812 \\
\hline
2\cdot6624782
\end{array}
= £460 \text{ as before.}
$$

(C)

$$
\begin{array}{r}
\cdot0138901 \\
30 \\
\hline
\cdot4167030 \\
\text{Log. } 1 = \cdot0000000 \\
\hline
\bar{1}\cdot5832970
\end{array}
= \cdot383
$$

£1200 × ·383 = £460 as before.

Ex. 5.—The present value at 7 per cent. of £2100 due in 15 years = £760.

Ex. 6.—The present value at 4½ per cent. of £700 due in 18 years = £317.

The present value is the sum which will amount in the time at the stated rate of interest to the sum named (Prob. 1). Hence dividing the sum named by the amount of £1 will give the present value of the sum named.

PROB. 3. To find the number of years in which a given sum will amount to any other sum.

Find from the table of logs. the log. of the stated amount and the log. of the given sum, and subtract the latter from the former; the remainder will be the log. of the amount of £1 in the time. Divide this remainder by the log. of the amount of £1 in one year (Aux. Table); the quotient will be the number of years required.

Ex. 7.—In how many years will £2000 double itself at 5 per cent. interest?

$$\begin{array}{rl} \text{Log. } 4000 = & 3{\cdot}6020600 \\ \text{Log. } 2000 = & 3{\cdot}3010300 \\ \hline {\cdot}0211893) & {\cdot}3010300(14 \\ & 211893 \\ \hline & 891370 \end{array}$$

Ans. In rather more than fourteen years.

Ex. 8.—The time required for £759 to amount to £2360 at 3½ per cent. = 33 years.

Ex. 9.—The time required for £4000 to amount to £6000 at 7 per cent. = 6 years.

The subtraction of logarithms in this rule is tantamount to dividing the stated amount by the given sum, the quotient of which is evidently the amount of £1; so that the entire rule is the inverse of that for Prob. 1.

PROB. 4. To find the rate of interest at which a given sum accumulates in a certain number of years to a stated amount.

Find from the table of logs. the log. of the stated amount and the log. of the given sum. Subtract the latter from the former, and divide the remainder by the number of years. Refer to the Aux. Table, and find in the column of the logs. of the amount of £1 such log. as is nearest to the quotient produced as above; the corresponding rate of interest (in the preceding column) will be for almost all purposes a sufficiently near approximation to the actual rate. (If greater accuracy be required, the precise rate of interest can be obtained by ascertaining the natural number corresponding to the quotient above produced. Then subtract £1 from such natural number, and multiply the remainder by 100; the product will be the rate per cent. interest.)

Ex. 10.—The sum of £300 invested 12 years since is found to have amounted to £639; what rate of interest has been realized?

Or:—A person has purchased for £300 a perpetual rent-charge to be entered upon at the expiration of 12 years, and the value of which if in

possession he estimates to be £639. At what rate of interest may he consider he has invested his money?

$$
\begin{array}{rl}
\text{Log. } 639 = & 2{\cdot}8055009 \\
\text{Log. } 300 = & 2{\cdot}4771213 \\
\hline
12) & {\cdot}3283796 \\
\hline
 & {\cdot}0273649
\end{array}
$$

Ans. About $6\frac{1}{4}$ per cent.

Ex. 11.—The rate of interest at which £1000 will in 21 years amount to £2280 = 4 per cent.

Ex. 12.—The rate of interest at which £260 will in 6 years amount to £349 = 5 per cent.

This rule admits of the same explanation as that for Prob. 3.

ANNUITIES FOR TERMS OF YEARS.

AMOUNTS.

PROB. 5. To find the amount of a given sum per annum in any number of years.

Divide the interest which £1 would produce in the given time by the annual interest of £1; the quotient will be the amount of an annuity of £1. Multiply the given annual sum by this amount; the product will be the amount required. (The above-mentioned interest produced by £1 in the given time is the remainder left upon the subtraction of £1 from the amount of £1 ascertained as in Prob. 1.)

Ex. 13.—To what sum will an annuity of £90 accumulate in 22 years at 5 per cent. interest?

$$\begin{array}{r} \cdot 0211893 \\ 22 \\ \hline 0423786 \\ 423786 \\ \hline \cdot 4661646 \\ \hline \end{array} = \begin{array}{r} 2\cdot 925 \\ 1\cdot \\ \hline \cdot 05)\ 1\cdot 925 \\ \hline 38\cdot 5 \\ \hline \end{array}$$

£90 × 38·5 = £3465 *Ans.*

Ex. 14.—The amount of an annuity of £250 in 26 years at 3¼ per cent. = £250 × 40 = £10,000.

Ex. 15.—The amount of an annuity of £2000 in 7 years at 8 per cent. = £2000 × 8·92 = £17,840.

The above-mentioned interest produced by £1 in the time actually consists of the receipts by way of annual interest on the £1, and of the interest produced by them; in other words, it is the amount of an annuity equal to the annual interest of £1. Therefore, As an annuity consisting of the annual interest of £1 : the sum to which it will amount (viz., the interest produced by £1 in the time) : : an annuity of £1 : the amount of such last-named annuity. Therefore the interest produced by £1 in the time, divided by the annual interest of £1, will give the amount of an annuity of £1.*

If a table of the amounts (at any given rate of interest) of £1 in 1, 2, 3, &c., years respectively be calculated (Prob. 1), it would be easy to construct thence a corresponding *Table of the Amounts of an Annuity of* £1. Thus in the case of an annuity of £1 of which only *one* year's income remains to be received (at the

* Adapted from the reasoning in Mr. Milne's Treatise.

end of the year) the amount will be £1 only. In the case of the annuity of £1 continuing for *two* years, there is an additional year's income producing interest during one year of the term; therefore the amount of such annuity will be shown by the addition of the amount of £1 in *one* year (appearing in the first-mentioned table) to the amount of the last-named annuity. In the case of the annuity of £1 continuing for *three* years, there is again an additional year's income producing interest during two years of the term; therefore the amount of such annuity will be shown by the addition of the amount of £1 in *two* years to the amount of the last-named annuity: and so on. It is obvious that the amount of an annuity of £1 must be the sum total of the amounts of all the pounds received, in the periods between their respective receipts and the expiration of the annuity.

PROB. 6. To find what sum per annum will in a given number of years amount to a stated sum.

Multiply the stated sum by the interest of £1 in one year. Divide the product by the interest of £1 in the given time; the quotient will be the annuity required. (The interest of

£1 in the given time is the remainder left upon the subtraction of £1 from its amount ascertained as in Prob. 1.)

Ex. 16.—What sum must be annually invested in order to amount at the expiration of 16½ years to £5000, the interest obtained being 3½ per cent.?

$$
\begin{array}{r}
\cdot0149403 \\
16\tfrac{1}{2} \\
\hline
2390448 \\
0074702 \\
\hline
\cdot2465150
\end{array}
= \begin{array}{r}
1\cdot764 \\
1\cdot \\
\hline
\cdot764\,)
\end{array}
\quad
\begin{array}{|l}
5000 \\
\cdot035 \\
\hline
175\cdot000\ (£229\ \textit{Ans.} \\
\ \ 1528 \\
\hline
\ \ \ \ 2220 \\
\ \ \ \ 1528 \\
\hline
\ \ \ \ \ 6920
\end{array}
$$

Ex. 17.—The sum which must be annually invested at 10 per cent. to provide in 21 years £6400 = £100.

Ex. 18.—The sum which must be annually invested at 4½ per cent. to provide in 15 years £20,000 = £980.

This rule is a partial inversion of that for

Prob. 5, as may be seen from the respective detailed examples. It is equivalent to dividing the stated sum by the amount of an annuity of £1.

The quotient resulting from the division of the interest of £1 in one year by the interest of £1 in the time would be the *annual sinking fund* to be accumulated at compound interest, in order to provide £1 at the end of any number of years.

PROB. 7. To find the number of years which must elapse before a given annuity will amount to a stated sum.

Divide the stated sum by the annuity; the quotient will be the amount of an annuity of £1. Multiply this by the interest of £1 in one year, the product will be the interest of £1 in the given time. This augmented by £1, will be the amount of £1 in the given time, from which the number of years may be ascertained by finding (as in Prob. 3) the log. of such amount, and dividing this log. by the log. of the amount of £1 in one year (Aux. Table); the quotient will be the number of years required.

Ex. 19.—In how many years will an annual saving of £50, if invested at the end of each

year at 2¾ per cent. interest, amount to £1000?

$$
\begin{array}{l}
50)\underline{1000} \\
\quad\ 20 \\
\quad\ \underline{\cdot 0275} \\
\quad\ \cdot 55 \qquad \cdot 0117818)\cdot 1903317(16 \text{ yrs. } Ans. \\
\quad\ \underline{1\cdot} \qquad\qquad\qquad\quad \underline{117818} \\
\quad\ 1\cdot 55 = \cdot 1903317 \qquad \underline{725137}
\end{array}
$$

Ex. 20.—The time in which an annuity of £1000 will amount at 9 per cent. to £7523 = 6 years.

Ex. 21.—The time in which an annuity of £750 will amount at 4½ per cent. to £15,000 = 14 years and 7 months.

This rule is the inverse of that for Prob. 6, as may be seen from the respective detailed examples.

PROB. 8. To find the rate of interest at which a given annuity will accumulate in a certain number of years to a stated amount.

Divide the stated amount by the given annuity, the quotient will be the amount of an annuity of £1. The rate of interest may be approximated by making experiments from the

Aux. Table, to ascertain at which of the rates of interest there set out the above amount of an annuity of £1 will by Prob. 5 be the most nearly arrived at.

Ex. 22.—The rate of interest at which an annuity of £200 will in 14 years amount to £4200 is 6 per cent.

PRESENT VALUES.

I. *Perpetual Annuities.*

1. IMMEDIATE. Freehold estates in possession.

PROB. 9. To find the present value of a given perpetual annuity.

Divide 100 by the rate per cent. interest which it is assumed the property should yield; the quotient will be the value of an annuity of £1, or the "years' purchase" for any other annuity. Therefore, multiply the given annuity by such years' purchase; the product will be the present value required.

(The value of an annuity of £1 may, however, be ascertained by an inspection of the third column of the Aux. Table with reference to any of the rates of interest there set out.)

Ex. 23.—What is the present value at 4 per cent. interest of a perpetual annuity of £3400, or of a freehold estate producing annually that amount?

$$\begin{array}{l} 4\,)\,100 \\ \hline \;\;25 \end{array} \quad £3400 \times 25 = £85{,}000. \ \textit{Ans.}$$

25 years' purchase appears in the third column of the Aux. Table.

Ex. 24.—The present value at $5\frac{1}{2}$ per cent. interest of a perpetual annuity of £18,000 = £327,273.

Ex. 25.—The present value at 10 per cent. interest of a perpetual annuity of £789 = £7890.

The present value of such an annuity is the sum of money which, if invested at the specified rate of interest, will produce an annual income equal to the annuity. Thus £85,000 (Ex. 23), if invested at 4 per cent. interest, will produce £3400 per annum.

By obvious inversions of the above rule we may ascertain the *perpetual annuity* which any present sum will purchase, and the *rate of interest* by which any such purchase is calculated.

2. PERPETUAL ANNUITIES DEFERRED.—Freehold Estates in Reversion, &c.

PROB. 10. To find the present value of a given perpetual annuity which is deferred for a certain number of years.

Multiply the log. of the amount of £1 in one year at the stated rate of interest (Aux. Table) by the number of years for which the annuity is deferred. Subtract the product from the log. of the value of £1 per annum in perpetuity (Aux. Table). The natural number corresponding to the remainder will be the present value of a perpetual annuity of £1 (deferred as in the question), or the years' purchase for any other annuity; therefore multiply the given annuity by it, and the product will be the present value required.

EX. 26.—What is the present value at $3\frac{1}{2}$ per cent. interest of a perpetual annuity of £1000 to be entered upon at the expiration of 25 years?

Or:—What is the present value at $3\frac{1}{2}$ per cent. interest of the reversion of a freehold property producing £1000 per annum, but outstanding upon a beneficial lease, of which 25

years are unexpired? [In this case an addition must be made (strictly by Prob. 12) for the present value of any rent which will be receivable by the purchaser during the continuance of the lease.]

$$
\begin{array}{rrl}
 & \cdot0142404 & \\
 & 25 & \\
\hline
 & \cdot3560100 & \\
\text{Log. } 30 = & 1\cdot4771213 & \\
\hline
 & 1\cdot1211113 & = 13\cdot2 \\
\hline
\end{array}
$$

£1000 × 13·2 y.p. = £13,200 *Ans.*

Ex. 27.—The present value at 8 per cent. interest of a perpetual annuity of £4450 deferred 6 years = £4450 × 7·88 = £35,066.

Ex. 28.—The present value at 10 per cent. interest of a perpetual annuity of £300 deferred 14½ years = £300 × 2·51 = £753.

The present value of such a reversion is the sum which if invested at the particular rate of interest will by the end of the time named have amounted to the then value of the perpetual annuity which will be entered upon; and it is therefore to be obtained by discounting the latter value. Hence the above rule is an application of Prob. 2 (B) to the value of a per-

petual annuity of £1 (Prob. 9). The perpetuity of £1000 per annum (Ex. 26) if calculated at 3⅓ per cent. will be worth £30,000; and £13,200 will, by Prob. 1, amount, in 25 years, to £13,200 × 2·27 = about £30,000.

PROB. 11. To find the rate of interest by which the calculation of the present value of a given deferred perpetual annuity results in a stated sum.

From the log. of the stated sum subtract the log. of the annuity; the remainder will be the log. of the years' purchase. The rate of interest may be approximated by making experiments from the Aux. Table to ascertain which of the rates of interest there set out will, by Prob. 10, produce a "remainder" (as there mentioned), the nearest to the above remainder.

Ex. 29. The treaty for the sale of a perpetual annuity of £800 to be entered upon at the expiration of 12 years having resulted in the price of £2500 being agreed upon, what is the rate of interest?

$$\begin{array}{r} \text{Log. } 2500 = 3{\cdot}3979400 \\ \text{Log. } 800 = 2{\cdot}9030900 \\ \hline {\cdot}4948500 \\ \hline \end{array}$$

It will be found by experiments that 10 per cent. is the rate in the Auxiliary Table which will produce a remainder nearest to the above.

II.—*Temporary Annuities.*

1.—Immediate. Estates on beneficial leases, building leases, &c.

Prob. 12.—To find the present value at a stated rate of interest of a given annuity continuing for a certain number of years from the present time.

Find the present value of the reversion to a perpetual annuity of £1, if deferred for the number of years mentioned, thus (Prob. 10):—Multiply the log. of the amount of £1 in one year at the stated rate of interest (Aux. Table) by the number of years, and subtract the product from the log. of the present value of a perpetual annuity of £1 in possession (Aux. Table); the natural number corresponding to the remainder will be the present value of the said reversion. Deduct this present value from that of a perpetual annuity of £1 in possession (Aux. Table), and the remainder will be the present value of an annuity of £1 continuing for the stated number of years, or the "years' pur-

chase" for any other annuity; therefore multiply the given annuity by it, and the product will be the present value required.

Ex. 30.—What is the present value at 5½ per cent. of an annuity of £300 to be now entered upon, and to continue 31 years?

Or :—What premium or fine should be paid for the grant of a beneficial lease for 31 years (or for the purchase of a beneficial lease of which 31 years are unexpired) of property producing a net annual income, after deducting the rent payable under the lease, of £300, interest being calculated at 5½ per cent?

$$\begin{array}{r} \cdot0232525 \\ 31 \\ \hline \cdot7208275 \\ 1\cdot2596373 \\ \hline \cdot5388098 \end{array} = \begin{array}{r} 3\cdot46 \\ 18\cdot182 \\ \hline 14\cdot72 \\ 300 \\ \hline \pounds4416 \end{array} \quad \textit{Ans.}$$

Ex. 31.—The present value at 9 per cent. of an

annuity of £1700 for 9½ years = £1700 × 6·21 = £10,557.

Ex. 32.—The present value at 4½ per cent. of an annuity of £740 for 13 years = £740 × 9·68 = £7163.

If the present value of a perpetual annuity of £1, deferred for a certain number of years (Prob. 10), be deducted from the present value of a perpetual annuity of £1 enjoyed from the present time (Prob. 9), the remainder will be the present value of an annuity of £1 for the intermediate period—*i.e.*, of an annuity of £1 immediately entered upon, and continuing for the stated number of years. Hence the rule is deduced from the rules for the Problems above mentioned.

If a table of the present values, at any given rate of interest, of £1 due at the end of 1, 2, 3, &c. years, be calculated (Prob. 2), it would be easy to construct thence a corresponding *table of the present values of an annuity of* £1. Thus the present value of an annuity of £1, of which only *one* year's income remains to be received, is evidently the present value (appearing in the first mentioned table) of £1, due at the end of *one* year. By adding to this the present value

of £1, due at the end of *two* years, we evidently obtain the present value of an annuity of £1 continuing for *two* years. By adding to this the present value of £1, due at the end of *three* years, we evidently obtain the present value of an annuity of £1 continuing for *three* years. And so on. It is obvious that the present value of an annuity of £1 must be the sum total of the present values of £1, due at the end of each successive year for which the annuity is receivable.

The price computed by the rule for Prob. 12, as the present value of the annuity, is a sum which, if invested at the stated rate of interest, would, at the expiration of the time, be equal in amount (Prob. 1) to the sum to which the annuity itself would, if similarly invested year by year, have accumulated in the same time (Prob. 5). Thus (Ex. 30) £4416 × 5·26 = about £23,228, and £300 × 77·4 = very nearly the same.

Again. If out of the annual income the purchaser only take the stated rate of interest upon his purchase money, and invest the remaining income year by year, the latter will at the expiration of the time amount (Prob. 5) to a sum equal to the price he paid, provided he effect such re-investment at the rate of interest

adopted in the calculation of the price. Thus (Ex. 30) 5½ per cent. interest upon £4416 = £243; and the remaining income, £57, will amount in 31 years to £57 × 77·4 = very nearly £4416.

But it frequently happens that the surplus income *cannot be re-invested at so high a rate* of interest as that at which the price of the annuity has been calculated. In this case the purchaser must content himself with using a less rate of interest upon his purchase money if he wish to replace the latter by the end of the time. To ascertain what is the rate of interest which may be thus realized, he can find by Prob. 6 how much of the income must be annually devoted to investment for replacing the capital; the remaining income will be available interest. The rate per cent. interest which this remaining income is to the purchase money may of course be ascertained by multiplying it by 100, and dividing the product by the purchase money; the quotient will be the rate per cent. interest realized.

PROB. 13.—To find the sum of money which should be given for the purchase of an annuity for a certain number of years in order that the

purchaser may be able yearly to avail himself of a stated rate of interest upon the purchase-money, and also to have his capital replaced at the expiration of the time by annual investments of the surplus income at some other rate of interest.

The value of an annuity of £1 under such circumstances, or the "years' purchase" for any other annuity will be the quotient produced by the following means :—

Dividend :—100 times the amount (Prob. 5) of £1 per annum in the time at the investment rate of interest.

Divisor :—100, added to the product of the available rate per cent. interest when multiplied by the said amount of £1 per annum.

Therefore multiply the given annuity by such quotient, and the product will be the sum required.*

* Assuming an annuity of £1, as above,—

Let r be the rate per cent. interest desired to be realized ;

Then $\frac{r}{100}$ will be the interest to be produced by every pound of the purchase money.

Let A be the amount of £1 per annum in the stated time at the investment rate of interest.

Let x be the sought price for the annuity of £1.

$$\text{Then } x = \left(1 - \frac{x\,r}{100}\right) A = A - \frac{A\,x\,r}{100}$$

$$x + \frac{A\,x\,r}{100} = A\,;\ 1 + \frac{A\,r}{100} = \frac{A}{x}$$

$$x = \frac{100\,A}{100 + A\,r}$$

Ex. 33.—A person wishes to bid at an auction for the purchase of the unexpired term of 22 years of the beneficial lease of a property producing a net income (after deduction of the rent reserved in the lease) of £250. He is desirous of receiving during the continuance of the lease 7 per cent. interest upon his outlay in the purchase, but he will only be able to invest the surplus income (remaining after the deduction of such interest) at 5 per cent. for the replacement of his capital at the expiration of the lease. To what extent may he bid for the purchase?

By Prob. 5 (Ex. 13). The amount of £1 per annum in 22 years at 5 per cent. is £38·5, so the dividend will be £3850.

The divisor will be

£38·5 × 7 = 269·5
Add 100·

369·5) 3850 (10·4
3695
15500

£250 × 10·4 = £2600

The available interest acquired is 7 per cent. upon £2600 = £182 per annum. The remain-

ing income, £68, if invested year by year at 5 per cent., will amount (Prob. 5) at the expiration of 22 years to £68 × 38·5 = about £2600, thus replacing the purchase-money.

Ex. 34.—The sum which should be given for the purchase of an annuity of £5600 for 14 years, so that the purchaser may make 5 per cent. interest, and may replace his capital by means of investments at 3½ per cent. interest = £5600 × 9·38 = £52,528.

Prob. 14.—To find the annuity which any given sum will purchase, to continue for a stated number of years from the present time.

Divide the given sum by the "years' purchase" ascertained by Prob. 12 (or Prob. 13 if *it* should be assumed as the basis). The quotient will be the annuity required.

The reason of this rule is evident.

Prob. 15.—To find the rate of interest by which the calculation of the present value of an immediate annuity results in a stated sum.

Divide the sum by the annuity; the quotient will be the "years' purchase." The rate of interest may be approximated by making experi-

ments from the Auxiliary Table to ascertain which of the rates of interest there set out will, by Prob. 12, give a "years' purchase" value (as there mentioned), the nearest to the quotient above produced.

Ex. 35.—The sum of £13,950 was paid for the purchase of a beneficial lease for 14 years, of a property estimated to produce (after deducting the rent reserved in the lease) £1500 per annum; therefore the rate of interest was 6 per cent.

2.—TEMPORARY ANNUITIES, DEFERRED. Concurrent Beneficial Leases.—Renewals or Extensions of Beneficial Leases, &c.

PROB. 16.—To find the present value of a given annuity commencing at the expiration of a stated period, and continuing for a certain number of years.

Find by Prob. 10 the present values of a perpetual annuity of £1 deferred for the two periods respectively; the difference between them will be the value of an annuity of £1 for the intermediate period, or the "years' purchase" for any other annuity. Therefore multiply the

given annuity by this, and the product will be the present value required.

Ex. 86.—What is the present value at 7 per cent. interest of a deferred annuity of £400, commencing after the lapse of 12 years, and then to continue 28 years?

Or:—A property is now held upon a beneficial lease by A., who declines to renew his lease, of which 12 years are unexpired. Assuming the property to yield (after deducting the rent payable under the lease) a net income of £400 per annum, what sum should, at the rate of 7 per cent. interest, be paid by B., as a fine or premium, for a concurrent beneficial lease for 40 years, of which only 28 will therefore be productive to him?

Or:—What fine, calculated at 7 per cent. interest, should be paid by a lessee for the renewal of 28 years lapsed in a beneficial lease, originally granted for 40 years, of property producing (after deducting the rent payable under the lease) a net income of £400 per annum?

·0293838	·0293838
12	40
·3526056	1·1753520
1·1549020	1·1549020
·8022964 = 6·34	$\bar{1}$·9795500 = ·954

6·34
·95

5·39 × £400 = £2156. *Ans.*

Ex. 37.—The present value at 10 per cent. of an annuity of £1250 deferred 14 years, and then continuing 7 years, is £1250 × 1·28 = £1600.

Ex. 38.—The present value at 4½ per cent. of an annuity of £760, deferred 13 years, and then continuing 20 years, is £760 × 7.3 = £5548.

The above rule explains itself. It is evident that the renewal fine produced in this problem, is the same as the difference between the present values of the existing lease and of the new lease, respectively, if separately calculated according to Problem 12. The latter mode of calculation should be adopted if the rent to be reserved in the new lease will differ from that in the existing lease.

The price thus computed as the present value of a deferred annuity, is a sum which if now invested at the stated rate of interest, will, at the expiration of the annuity, be equal in amount (Prob. 1) to the sum to which the annuity itself, when entered upon, would, if similarly invested year by year, have accumulated, (Prob. 5.) Thus, taking Ex. 36, it may be seen that the amount of £2156 in 40 years, and the amount of £400 per annum in 28 years, are each equal to about £32,300.

This shows that by investing at the rate of interest adopted in any calculation as above, the purchaser will be able out of the income to replace his outlay or capital, and will also obtain an equivalent for interest at the said rate, upon his outlay, for the entire period between payment and the expiration of the annuity.

With respect to a simple purchase of a deferred annuity, or of a concurrent beneficial lease, the portion of the income which must, when entered upon, be annually invested in order to replace the purchase-money by the expiration of the annuity or lease, can be ascertained by Prob. 6.

With respect to renewals of a beneficial lease already in possession,—Assume a person to purchase in the first instance a lease for a cer-

tain number of years. (Prob. 12.) If he only take from the annual income interest upon the sum paid at the rate upon which the calculation was based, and annually invest the remaining income at the same rate of interest, the accumulated amount of the latter (Prob. 5) will at any time be equal to the fine or premium which, if calculated (Prob. 16) upon the same data as before, would obtain for him at such time a renewal of the lapsed portion of his lease, *i.e.*, an extension of his lease to the full term originally granted; his original capital will thus always be preserved.

PROB. 17.—In the case of the renewal of a beneficial lease, to ascertain the amount of rent which might be paid from the present time during the term of the new lease, in lieu of renewing by payment of a premium or fine.

The lessee is now entitled to the present value of the existing lease. This sum must therefore be converted into an equivalent annuity (Prob. 14), continuing from the present time till the expiration of the new lease; deduct such annuity from the annual value of the property, and the remainder will be the rent to be paid over by the lessee.

If a certain sum is agreed to be paid as a fine,

and it is required to ascertain what rent should be paid as the *remainder* of the consideration for the renewal, this sum can be added to the present value of the existing lease, and the total be then converted into an equivalent annuity as above.

PROB. 18.—To find the rate of interest by which the calculation of the present value of a deferred annuity results in a stated sum.

Divide the sum by the annuity; the quotient will be the "years' purchase." The rate of interest may be approximated by making experiments from the Auxiliary Table to ascertain which of the rates of interest there set out will, by Prob. 16, give a "years' purchase" (as there mentioned) the nearest to the quotient above produced.

Ex. 39.—Assuming that the sum of £1055 was paid for the purchase of an annuity of £280, deferred for 9 years, and then to continue 12 years (or for the purchase of a concurrent lease, or the extension of a lease, for the above period,) the rate of interest adopted in the calculation will be found to have been 8 per cent.

ANNUITIES FOR A LIFE.

TABLE OF THE PRESENT VALUES,

*At 3, 4, and 5 per Cent. Compound Interest per annum, of an Annuity of £1, continuing for the term of a Life, according to the English Life Table, No. 2 (Males.)**

These values are taken from a table in the Appendix to the Twelfth Annual Report of the Registrar-General of Births, Deaths, and Marriages in England, who thus alludes to it in his report :—

"The Appendix contains a new English Life "Table, carefully calculated by Mr. Farr. It is "on a more extended basis than the Life Table "published in my Sixth Annual Report, being "founded on the ages at which the entire num- "ber of deaths were registered which occurred in "England and Wales in the *seven* years ending "1844. The Life Table published in my Sixth "Report was founded on the facts recorded for

* Tables upon the above basis have been calculated by Mr. W. A. Yool of the values of annuities on single and joint lives at £3. 6s. 8d. per cent

"*one* year, 1841 : but the variation in the results "is very trifling."

The values are those of an annuity of £1 payable at the end of the first and of every subsequent year.

Age.	3 per cent.	4 per cent.	5 per cent.	Age.
	£	£	£	
0	18·2660	15·0764	12·7584	0
1	21·3827	17·6535	14·9374	1
2	22·5635	18·6429	15·7805	2
3	23·1106	19·1146	16·1899	3
4	23·4061	19·3821	16·4295	4
5	23·5587	19·5340	16·5733	5
6	23·5969	19·5928	16·6396	6
7	23·5502	19·5823	16·6480	7
8	23·4679	19·5429	16·6326	8
9	23·3481	19·4728	16·5915	9
10	23·2042	19·3828	16·5339	10
11	23·0307	19·2680	16·4553	11
12	22·8359	19·1353	16·3613	12
13	22·6331	18·9956	16·2612	13
14	22·4226	18·8490	16·1552	14
15	22·2225	18·7109	16·0564	15
16	22·0035	18·5566	15·9434	16
17	21·7928	18·4089	15·8361	17
18	21·5962	18·2728	15·7387	18
19	21·4141	18·1490	15·6519	19
20	21·2334	18·0263	15·5663	20
21	21·0490	17·9005	15·4781	21
22	20·8607	17·7712	15·3871	22
23	20·6682	17·6383	15·2930	23
24	20·4714	17·5017	15·1959	24
25	20·2703	17·3612	15·0955	25
26	20·0646	17·2167	14·9916	26
27	19·8542	17·0680	14·8841	27
28	19·6391	16·9150	14·7730	28
29	19·4192	16·7576	14·6580	29
30	19·1943	16·5957	14·5391	30

Age.	3 per cent.	4 per cent.	5 per cent.	Age.
	£	£	£	
31	18·9646	16·4293	14·4161	31
32	18·7298	16·2581	14·2891	32
33	18·4899	16·0822	14·1577	33
34	18·2450	15·9015	14·0220	34
35	17·9951	15·7159	13·8819	35
36	17·7400	15·5254	13·7373	36
37	17·4799	15·3300	13·5881	37
38	17·2147	15·1294	13·4342	38
39	16·9443	14·9238	13·2756	39
40	16·6689	14·7131	13·1120	40
41	16·3883	14·4971	12·9435	41
42	16·1026	14·2759	12·7700	42
43	15·8118	14·0493	12·5913	43
44	15·5158	13·8173	12·4073	44
45	15·2146	13·5798	12·2178	45
46	14·9080	13·3366	12·0227	46
47	14·5962	13·0876	11·8219	47
48	14·2788	12·8327	11·6151	48
49	13·9559	12·5717	11·4021	49
50	13·6272	12·3044	11·1826	50
51	13·2925	12·0304	10·9563	51
52	12·9517	11·7495	10·7229	52
53	12·6043	11·4614	10·4819	53
54	12·2501	11·1656	10·2330	54
55	11·8886	10·8617	9·9754	55
56	11·5193	10·5490	9·7086	56
57	11·1455	10·2304	9·4351	57
58	10·7780	9·9159	9·1641	58
59	10·4155	9·6044	8·8946	59
60	10·0572	9·2951	8·6260	60

Age.	3 per cent.	4 per cent.	5 per cent.	Age.
	£	£	£	
61	9·7027	8·9877	8·3579	61
62	9·3515	8·6819	8·0899	62
63	9·0037	8·3775	7·8222	63
64	8·6593	8·0748	7·5547	64
65	8·3185	7·7738	7·2876	65
66	7·9817	7·4751	7·0214	66
67	7·6493	7·1790	6·7565	67
68	7·3218	6·8859	6·4932	68
69	6·9998	6·5966	6·2322	69
70	6·6837	6·3114	5·9739	70
71	6·3742	6·0311	5·7191	71
72	6·0718	5·7561	5·4682	72
73	5·7770	5·4869	5·2218	73
74	5·4902	5·2242	4·9804	74
75	5·2119	4·9683	4·7445	75
76	4·9424	4·7197	4·5145	76
77	4·6820	4·4786	4·2909	77
78	4·4309	4·2455	4·0738	78
79	4·1893	4·0204	3·8638	79
80	3·9574	3·8037	3·6608	80
81	3·7350	3·5953	3·4652	81
82	3.5223	3·3954	3·2771	82
83	3·3190	3·2039	3·0965	83
84	3·1253	3·0208	2·9235	84
85	2·9407	2·8460	2·7579	85
86	2·7653	2·6792	2·5997	86
87	2·5986	2·5206	2·4489	87
88	2·4406	2·3699	2·3052	88
89	2·2910	2·2267	2·1691	89
90	2·1492	2·0907	2·0403	90

The values in the foregoing table claim especial attention in consequence of their having been deduced from materials recently collected throughout England and Wales; whereas the tables in common use are based upon imperfect observations made upwards of seventy years since in single towns. It is to be wished that tables were constructed upon the English basis of the present values, at various rates of interest, of annuities and reversions dependent upon one or more lives; meanwhile, for individual cases, we are able to estimate such values with as much accuracy as is essential for actual dealings with property.

APPENDIX.

SUGGESTIONS FOR THE MORE EQUITABLE ASSESSMENT OF THE INCOME-TAX.

A, being the possessor of an annuity of £100, which will endure for 31 years, if he invest it year after year at 3 per cent. compound interest, it will amount at the end of the term to £5000. Therefore, the sum of ready money, which B should pay to A for the purchase of his interest in the annuity, is the sum which, if put out at 3 per cent. compound interest, will at the end of the 31 years also amount to £5000. Now, £1 will, under these circumstances, amount to £2 10s.; therefore, the purchase-money will be £2000, or 20 years' purchase of the annuity.

B having become the purchaser of the annuity, will receive every year £100. The proper interest upon his purchase-money (viz., £2000) will be only £60; and he should not use the remaining £40 as if it were income, but should invest it year by year, and these investments will at the expiration of the 31 years have amounted to £2000, thus reproducing his capital or original outlay.

According to the present assessment of the income-tax, however, B pays upon the entire annual pro-

duce. Assuming the tax to be, for facility of illustration, as high as one shilling in the pound, the amount he contributes is £5 per annum; and the present value of this payment for the period of the annuity is £5 multiplied by 20 years' purchase, or £100. But if he had invested the same amount of capital, viz., £2000, in the purchase of a *perpetual* annuity, the annual produce—in this case purely interest—would have been £60, upon which he would therefore only have had to pay £3 per annum for income-tax; and the present value of a perpetual annual payment of this amount is £3, multiplied by 33⅓ years' purchase, or £100 also. The startling fact is that in the case of any temporary annuity which a man may purchase—a life provision, for example, for a member of his family—the present value of the income-tax, which, under the existing system, will be paid during the continuance of the annuity, *however short it may be*, is actually as much as the present value of the entire income-tax which he and his descendants would have to pay if he invested the same amount of capital in the purchase of an annuity lasting for ever!

In order to produce correspondence with the taxation of perpetual annuities, the possessor of a temporary annuity ought only to be required to pay upon the portion of the produce which is actually interest; but the proportion which this bears to the whole varies according to the length of the term purchased, being a large portion in

long terms, but a very small portion in short terms, and in the latter cases, therefore, the inequality is severely felt. The difficulty appears to be that of finding a practical scale by which the portion of the annual produce which is actually interest shall be distinguished from the remaining portion which represents capital. The actuaries who were examined before the Committee of the House of Commons in 1852, appear to have regarded the subject with strict devotion to theory, and to have agreed as the basis of assessment that incomes should be capitalized *in detail*, having regard to their respective liabilities and to the length of time for which they would be enjoyed, and that a return should be made of capital also. The Commissioner of Inland Revenue to whom these opinions were referred, considered that such a plan must necessarily be more inquisitorial, and therefore more objectionable, than the present system, be more open to fraud, and involve an amount of extra labour in the calculation of the tax, which would render the scheme impracticable.

But if the extreme of accuracy is unattainable, is there no alternative but the extreme inaccuracy of the present system? The remedies of every-day life for difficulties of a similar character are to split differences, take averages, compromise in various ways. The results never precisely meet the views or the cases of all parties concerned, but they are allowed to be the only expedient course. Might not this treatment be applied to the income-tax?

Taking an average of years from 20 to 50, and this appears to be a fair range for averaging the original terms of temporary annuities, it will be found that, at the rate of 3 per cent. compound interest, the portion of produce which it is requisite annually to invest for the reproduction of capital, and which is therefore not properly taxable as income, averages one-third of the whole. Why not, therefore, let all such temporary annuities (with the exception of any granted for terms of years, say, exceeding 50) be taxed to the extent of only *two-thirds* of the rate imposed upon incomes from estates held in fee? This adjustment would not remove the inequalities in the taxation of terminable annuities as compared with each other, but it would do away with the general inequality complained of in the taxation of such annuities as compared with that of annuities in perpetuity. It would be an amelioration of the present system which not only similarly fails to recognise any distinction amongst terminable annuities, but involves the further and more striking error of making no allowance whatever for the circumstance that a portion of every such annuity is virtually capital.

Distinct, however, from the question of length of tenure, there are differences in the characters of properties which affect the incomes produced by them, whether terminable or in perpetuity. Incomes derived from funded property are at an advantage as compared with rents of the same amounts derived from lands, or from the various

classes of house property, inasmuch as such rents are subject to reduction by legal charges, losses, and the cost of reinstating the premises after the lapse of time. An allowance in respect of these circumstances may be made most simply and naturally in fixing the *amount of assessment;* and there would be no serious differences among professional men in fixing certain rates of deduction, from rent, for respective classes of such property.* Royalties upon brick-earth, and minerals, which now constitute a vexed question, might from their nature be reasonably taxed upon the two-thirds scale.

Turn now to industrial incomes. The witnesses before the Committee above mentioned differed widely, as might have been anticipated, with respect to the mode of treatment of such cases. As regards the amount of taxation, for instance, one thought that all industrial incomes should be taxed at the same rate as incomes from freehold property, whilst another considered that allowances should be made to such an extent as would indicate that professional incomes should be capitalized at seven years' pur-

* It is true that in calculations for the transfer of property allowance for such circumstances is customarily made by lowering the number of years' purchase at which the freehold is valued; but an equal allowance could be made (perhaps more naturally and certainly not more arbitrarily) by making a deduction from the lettable value, instead of by reducing the years' purchase, and such a practice might have precluded the questions which have arisen as to the rate of interest that should be adopted in an equitable calculation for the enfranchisement of leasehold house property.

chase, which would be virtually taxing them at less than a fourth of the rate imposed upon incomes from freehold property. A compromise between these views, and one not too favourable to industrial incomes, may perhaps be arrived at by an appropriate application to their case of the precise treatment that has been above suggested in respect of terminable annuities. For it will surely be only acknowledging a fact, and not granting a favour, to concede that the production of an industrial income exhausts what may be termed its capital, viz., the body and brain of the individual, and the cost of his education and support; and therefore, will it be asking too much that the earnings of a life may be treated in the same manner as that before proposed for terminable annuities from property, in which the capital embarked in them is similarly included? The range of years will not however be 20 to 50 as before; but, having regard to the uncertainty of life, and to the utmost length of time that a man is capable of earning his living, it appears right to substitute a range of years extending from 1 to 50. Now in the case of temporary annuities of this range, the portion of annual produce which (allowing 3 per cent. compound interest) represents capital, and therefore should not be taxed as income, is, on the average, one-half; and consequently the rate of tax upon industrial incomes should only be *one-half* of that charged upon incomes from freehold property.

II. The foregoing is an attempt to show that if the tax is charged upon only the income of freehold property, then, in order to relieve its present unjust pressure upon temporary annuities and industrial incomes, it must not be charged in their cases upon certain specified portions of annual produce, since these are virtually capital. But if objection can be made to the suggestion of viewing incomes from personal exertion as comprising the same elements as temporary annuities from property, and it be insisted that such incomes are simply income, it seems possible to show equally from this statement of the case that upon earnings, as compared with perpetual annuities, there should only be imposed the same half-scale to which the range of years in the previous argument has happened to point. The distinction which the present system does not recognise is, that in the one case there is capital which produces the income, and in the other case (according to the present argument) there is not. But it would seemingly only make the tax equitable and defensible upon its own merits if, when a portion is abstracted from every sovereign that is earned, the same portion should be abstracted not only from every sovereign which is received as interest, but also from every sovereign which constitutes capital, and by this course the above-mentioned distinction would be recognised. Now, in the case of property held in perpetuity, capital is equal to the value in ready money of the income or interest receivable year after year; therefore, in order to abstract (by

means of an annual tax) the above portion of every sovereign of capital, the interest must be taxed year after year with a duplicate rate—that is, if sixpence, for example, be deducted from every pound that is earned, one shilling must be deducted from every pound that is received as interest from freehold property—viz., sixpence to be taxed upon it as being interest, and sixpence to be taxed upon it as being an annuity equivalent to the capital. Similarly the tax upon temporary annuities should, upon the scale just assumed, be two rates of fourpence each; for a sixpenny rate should be charged upon two-thirds of the annual produce, this portion being (as stated in the first instance), upon an average, really interest, and a second sixpenny rate should be charged upon such portion as a tax upon the capital invested, with respect to which such interest is as a corresponding perpetual annuity. In other words, the entire annual produce should be charged eightpence in the pound, which bears to the one shilling rate upon perpetual annuities the two-thirds ratio proposed in the first division of this paper.

Possibly this mode of taxation is not too intricate, nor the views upon which it is founded too extreme, to allow of its being agreed upon as a compromise between conflicting interests.

THE END.

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